Capacitor Charge

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Introduction

Electronic devices are often ideal for displaying mathematical relationships. One application is to use the charging and discharging of a capacitor to show exponential decay. We will construct a circuit and measure the voltage across a charging and discharging capacitor. We will then compare the curves obtained experimentally with the ideal curves predicted from the known values of the resistance of the circuit, $R,$ and the capacitance, $C,$ of the capacitor.

This lab uses a CBL Voltage Probe to measure voltages. The data can easily be collected in 15 to 20 minutes and distributed to the students for their analysis and conclusions. This experiment offers an excellent opportunity to explore exponential decay. Knowledge of Algebra is a prerequisite.

Setup

Equipment Required:

CBL unit

TI-89 graphics calculator with a unit-to-unit link cable

$100\,k\Omega$ resistor, $\boldsymbol{R}$

$4.7\,\mu F$ capacitor, $\boldsymbol{C}$

one or two $1.5\,V$ batteries

wire

electric switch

Use the electronic components to construct the circuit shown in figure 1.

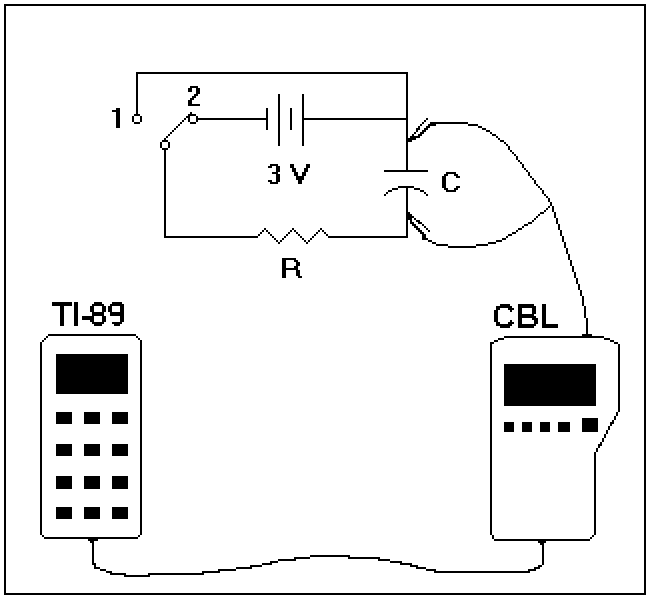

Figure 1: Equipment Setup

Equipment Setup Procedure

Connect the equipment as shown by Figure 1:

1. Connect the CBL unit to the TI-89 calculator with the unit-to-unit link cable using the I/O ports located on the bottom edge of each unit. Press the cable ends in firmly.

2. Connect the Voltage Probe to Channel 1 on the top of the CBL unit.

3. Clip one Voltage Probe lead to each side of the capacitor in the circuit.

4. Turn on the CBL unit and the calculator.
The CBL system is now ready to receive commands from the calculator.

Program Listing

This experiment requires that you download or enter the **CHARGE.89P** and the **DISCHARG.89P** programs listed in the appendix into your TI-89 calculator.

EXPERIMENT 1: Charge the Capacitor

Collect the Data

1. Move the switch to position 1 to ensure that the capacitor is completely discharged.

2. Make sure the CBL is turned on. Start the program **CHARGE** on the TI-89. The program will prompt:

   Press ENTER to set
   baseline voltage

3. The TI-89 will now display:

   Base Voltage = .012345
   Press ENTER to collect
   data and then
   close the circuit

4. First press [ENTER] and then close the circuit by moving the switch to position 2.

5. After the data is collected, a plot of voltage (in volts) vs. time (in seconds) appears on the calculator screen. The plot should look similar to the one shown in Figure 2. Make a printout of this graph using TI-GRAPH LINK or save it as a PIC variable to be printed later. Attach this printout to your worksheet. Be sure to include appropriate scales and axis labels on the printout The data is saved in lists L_4 and L_2. It would be prudent to save these lists to lists with new names, perhaps C1 and C2, as subsequent experiments will erase L_4 and L_2.

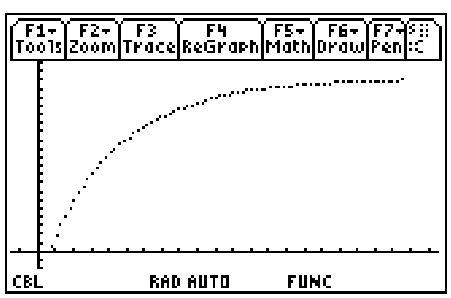

Figure 2.

Analyze the data

We know from physics that the voltage across a charging capacitor follows the equation:

$$V = V_0 * \left(1 - e^{-\frac{t}{RC}}\right)$$

We will attempt to match this exponential equation to the experimental data just obtained.

Procedure

1. From the screen shown in figure 2 press $GreenDiamond + [Y =]$ to get into the equation editor. Delete any existing equations. Now, as our first guess we will enter

$$y1(x) = 3*\left(1 - e^{-\frac{x}{.47}}\right)$$

Since in this example I am using two $1.5\,V$ batteries. Press $[GRAPH]$ to compare the graphs. My first attempt with this example data produced figure 3.

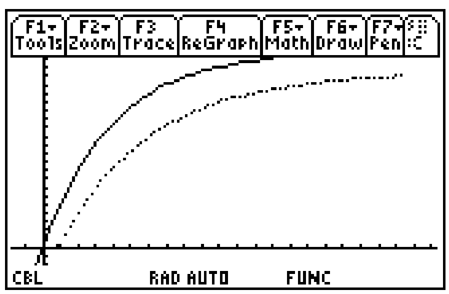

Figure 3.

2. Unless you are fortunate the fit could probably be improved. We will adjust some of the parameters to make a better fit.

Notice that the equation starts at the point $(0, 0)$ but my example experimental data does not. Using $[F3\,Trace]$ I can determine that my first "good" data point is $(0.12,\, 0.0398)$. I need to translate the equation graph in the positive x dimension by $+\,0.12$. I adjust the equation as shown below in the equation editor and then Press $GreenDiamond + [GRAPH]$ to check the fit.

$$y1(x) = 3*\left(1 - e^{-\frac{(x-0.12)}{.47}}\right)$$

Another difference between the data and the graph appears to be that the maximum voltage of my batteries was not the expected $3\,V$ but something less. Using $[F3\ Trace]$ again I can trace the data and determine that a maximum voltage of $2.7\,V$ is more realistic.

$$y1(x) = 2.7{*}\left(1 - e^{-\frac{(x-0.12)}{.47}}\right)$$

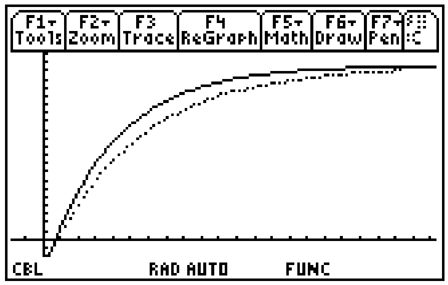

Figure 4.

Now we need to adjust the rate of increase of the exponential graph to fit the experimental data. This parameter can be adjusted by the value of $R{*}C$ in the denominator of the exponent. A larger value for this constant will decrease the rate of growth.

We could use a trial and error method but lets use the data and be more precise. Exponential decay has a half life. The time that the capacitor takes to charge half way can be found by tracing the data. Half way between $2.7\,V$ and $0.0398\,V$ is $1.33\,V$. This voltage occurs at time, $t = 0.52 - 0.12 = 0.4\,s$.

Since my exponential constant, $\frac{1}{RC}$, must equal $-\frac{1}{t}\ln\frac{1}{2}$, we will use $RC = 0.577$.

$$y1(x) = 2.7{*}\left(1 - e^{-\frac{(x-0.09)}{0.577}}\right)$$

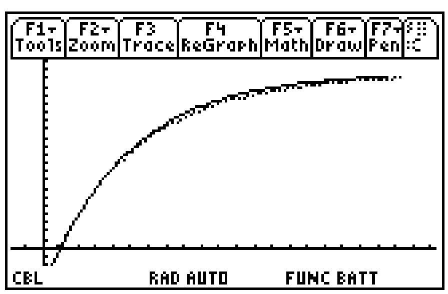

Figure 5.

The fit or the calculated curve to the data now appears convincing.

Analysis and Conclusion

1. By fitting the exponential curve to the data we have determined that the best value for the product of the circuit resistance, $\boldsymbol{R}$, and the capacitance, $\boldsymbol{C}$, is not 0.47 as we would have supposed, but due to error in the manufacturing process (within allowable tolerance), the values for the resistance and capacitance give a product of 0.577 instead. If we could precisely determine one value then the other would be precisely known.

2. It is also apparent that the advertised voltage output of the batteries is subject to variation and that parameter affects the overall performance of the circuit.

3. The translation in x is not physically significant. It is an anomaly of the way the data was gathered. But we still needed to account for the horizontal shift in the data to get a good fit.

EXPERIMENT 2: Discharge the Capacitor

Collect the Data

1. Move the switch to position 2 to ensure that the capacitor is completely charged.

2. Make sure the CBL is turned on. Start the program **DISCHARG** on the TI-89. The program will prompt:

> **Press ENTER to set**
> **baseline voltage**

3. The TI-89 will now display:

> **Base Voltage = 2.712345**
> **Press ENTER to collect**
> **data and then**
> **open the circuit**

4. First press [ENTER] and then open the circuit by moving the switch to position 1.

5. After the data is collected, a plot of voltage (in volts) vs. time (in seconds) appears on the calculator screen. The plot should look similar to the one shown in Figure 6. Make a printout of this graph using TI-GRAPH LINK or save it as a PIC variable to be printed later. Attach this printout to your worksheet. Be sure to include appropriate scales and axis labels on the printout The data is saved in lists L_4 and L_2. It would be prudent to save these lists to lists with new names, perhaps D1 and D2, as subsequent experiments will erase L_4 and L_2.

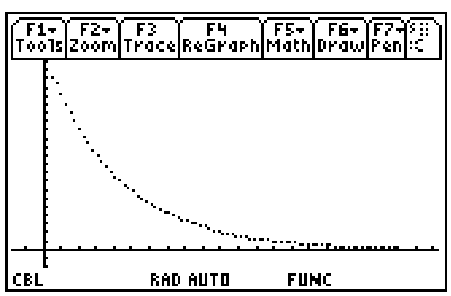

Figure 6.

Analyze the data

We know from physics that the voltage across a discharging capacitor follows the equation:

$$V = V_0 * e^{-\frac{t}{RC}}$$

We will attempt to match this exponential equation to the experimental data just obtained.

Procedure

1. From the screen shown in figure 2 press $GreenDiamond + [Y =]$ to get into the equation editor. Delete any existing equations. Now, as our first guess we will enter

$$y1(x) = 2.7 * e^{-\frac{x}{0.577}}$$

Press $[GRAPH]$ to compare the graphs. My first attempt with this example data produced figure 3.

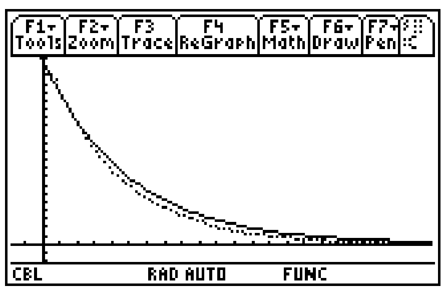

Figure 7.

2. With the information we gained from EXPERIMENT 1 we are able to get a reasonable fit right from the beginning. We will still adjust some of the parameters to make the best possible fit.

Notice that both the equation graph and the data plot start at the point $(0, 2.7)$. The battery output voltage is holding steady.

It appears that we could adjust the rate of decrease of the exponential graph to improve the fit to the experimental data. This parameter can be adjusted by the value of $R*C$ in the denominator of the exponent. A smaller value for this constant will increase the rate of decay. After several tries I determine that a value of 0.52 appears to give a good fit.

$$y1(x) = 2.7 * e^{-\frac{x}{0.52}}$$

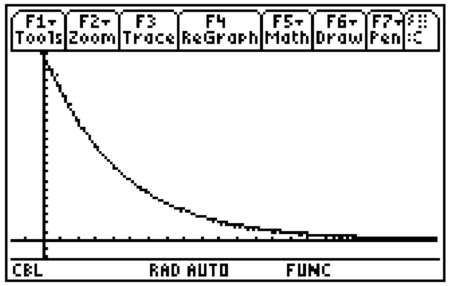

Figure 8.

Analysis and Conclusion

We have determined that the best value for the product of the circuit resistance, R, and the capacitance, C, is not 0.6 as we calculated in the first experiment but the values for the resistance and capacitance give a product of 0.52 instead. This is satisfactory verification of our previous results considering that we are not using precision electronic equipment to gather the data!

CHARGE

```
Prgm
80»n
.03»v
{3,v,n,2,1,1.5*q,6}»l1
ClrHome
ClrDraw
ClrIO
Send {0}
Send {1,0}
Send {1,1,2}
newList(n)»l2
newList(n)»l4
newList(5)»l6
Disp "Press ENTER to set"
Disp "baseline voltage"
Pause
Send {3,.01,5,0}
Get l6
(sum(l6)-max(l6)-min(l6))/3»q
ClrIO
Disp "Base Voltage = ",q
Disp "Press ENTER to collect"
Disp "data and then"
Disp "close the circuit"
Pause
Send l1
For i,1,n
v*i»l2[i]
EndFor
Get l4
l4-min(l4)»l4
NewData stat,l2,l4
NewPlot 1,1,l2,l4,,,,5
.1»xscl
1»yscl
ZoomData
DelVar l1,l6,n,i,v,q
EndPrgm
```

DISCHARG

```
Prgm
80»n
0.03»v
{3,v,n,3,1,0.9*q,5}»l1
ClrHome
ClrDraw
ClrIO
Send  {0}
Send  {1,0}
Send  {1,1,2}
newList(n)»l2
newList(n)»l4
newList(5)»l6
Disp  "Press ENTER to set"
Disp  "baseline voltage"
Pause
Send  {3,0.01,5,0}
Get  l6
(sum(l6)-max(l6)-min(l6))/3»q
ClrIO
Disp  "Base Voltage = ",q
Disp  "Press ENTER to collect"
Disp  "data and then"
Disp  "open the circuit"
Pause
Send  l1
For  i,1,n
v*i»l2[i]
EndFor
Get  l4
l4-min(l4)»l4
NewData  stat,l2,l4
NewPlot  1,1,l2,l4,,,,5
0.1»xscl
1»yscl
ZoomData
DelVar  l1,l6,n,i,v,q
EndPrgm
```