

Light Bulb Mathematics

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Introduction

This lab project is a collection of four experiments using the same equipment and setup. All four experiments can be run in one class period to collect the data. Data can then be distributed to the students for their analysis and conclusions. The four experiments explore the logistic function, the sine function, the inverse square function, and the exponential function. Knowledge of these functions is a prerequisite for this lab.

Setup

Because the setup of all four experiments is the same, it will be described here only once.

Equipment Required:

CBL unit

TI-89 graphics calculator with a unit-to-unit link cable

TI light probe

Lamp with a standard light bulb (60W to 150W)

Cardboard tube

Meter stick

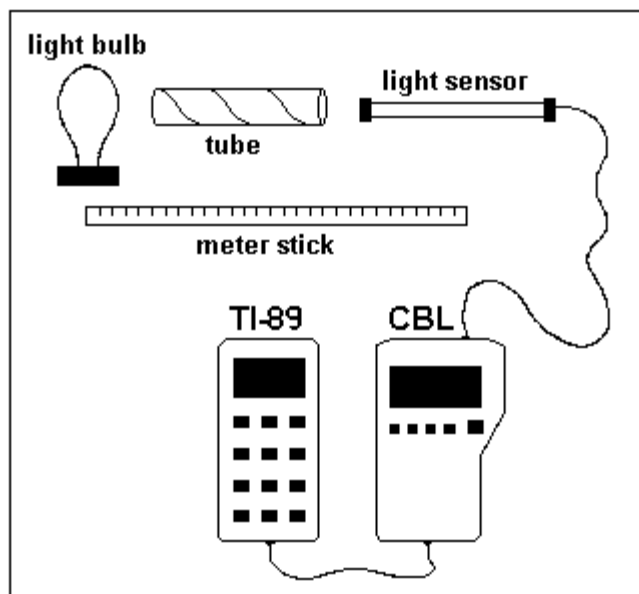


Figure 1: Equipment Setup

Equipment Setup Procedure

Connect the equipment as shown by Figure 1:

1. Connect the CBL unit to the TI-89 calculator with the unit-to-unit link cable using the I/O ports located on the bottom edge of each unit. Press the cable ends in firmly.
2. Connect the TI light probe to Channel 1 (CH1) on the top edge of the CBL unit.
3. Place the light sensor inside the cardboard tube. Position the tube and sensor so that the sensor is facing the light bulb as shown in Figure 1. The purpose of the tube is to keep out extraneous light. A long (30 in.) tube is best. This type of tube may be obtained from a roll of gift wrapping paper. If necessary, a short tube will work but the room lights will have to be dimmed and the tube must be carefully aimed at the light bulb for each data collection effort.
4. Turn on the CBL unit and the calculator.
The CBL system is now ready to receive commands from the calculator.

EXPERIMENT 1: Discovery

What happens when a light bulb is switched on? This experiment allows students to discover and investigate the mathematical curve that describes the light intensity increase of a light bulb when it is turned on.

Introduction

We generally refer to light intensity as "brightness." More precisely, intensity is defined as the rate at which energy is transferred per unit area, measured in Watts per square meter. When power is suddenly introduced to the filament of the light bulb, the filament heats until it glows, radiating the energy as light. We will use the TI light probe to discover the mathematical relationships of light intensity vs. time as a light bulb increases in brightness.

Program Listing

This experiment requires that you download or enter the **BULBON.89P** program listed in the appendix into your TI-89 calculator.

Experiment Procedure

1. Place the light sensor inside the cardboard tube and position the tube so that separation between the light sensor and the center of the bulb is between 20 in. and 30 in. If the sensor is placed too close to the bulb it will become saturated with light and the intensity curve will be unnaturally flattened at the high end.
2. Darken the room and turn off the light bulb or, if you have a long tube, place the end of the tube against the light bulb to block out as much light as possible.
3. Make sure the CBL is turned on. Start the program **BULBON** on the TI-89. The student who is positioning the light probe must carefully aim the probe at the light bulb by looking at the bulb through the tube. This position must now be held until the data is collected. If a long tube is used this care is unnecessary. When the program prompts to **PRESS [ENTER] TO ZERO** it will establish a baseline minimum light intensity.
4. The program will then prompt:
Base Voltage = .08654
Press ENTER to collect
data and then turn on
the light
5. After the data is collected, a plot of light intensity (in mW/cm^2) vs. time (in seconds) appears on the calculator screen. The plot should look similar to the one shown in Figure 2. Make a printout of this graph using **TI-GRAPH LINK** or save it as a **PIC** variable to be printed later. Attach this printout to your lab notebook. Be sure to include appropriate scales and axis labels on the printout. The data is saved in lists **L2** and **L4**. It would be prudent to save these lists to lists with new names, perhaps **D1** and **D2**, as subsequent experiments will erase **L2** and **L4**.

6. Notice that the data seem to show two different phenomena. The first is the long period tendency of the light to increase. The second is an apparent short period variation in the intensity with a smaller amplitude. This suggests two more experiments with the data collection parameters adjusted to highlight each of the different phenomenon in question.

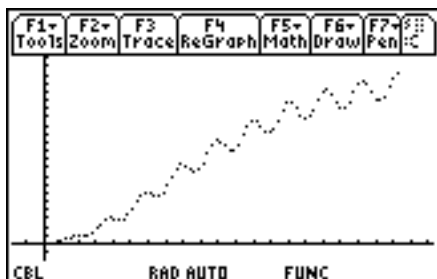


Figure 2.

EXPERIMENT 2: Bulb On

What is the long period behavior of the light intensity when a light bulb is switched on? This experiment allows students to discover and investigate the mathematical curve that describes the light intensity increase of a light bulb as it is turned on.

Introduction

The data collected in the **Discovery Experiment** showed an initial rapid rate of increase in light intensity followed by a decrease in the rate of increase as the light reached its full intensity. The curve is characteristic of quantities that will increase until available resources are being used as fast as they are being created. This relationship can be described by the logistic function

$$I(t) = \frac{C}{1 + Ae^{-Bt}}$$

Consequently, the intensity of the light increases almost exponentially when it is first turned on and then the rate of increase slows as the bulb reaches full output. In this experiment you will use the light intensity sensor to verify the relationship stated above.

Program Listing

This experiment requires that you download or enter the **BULBON.89P** program listed in the appendix into your TI-89 calculator.

Experiment Procedure

1. Modify the program **BULBON** to collect the long period data. Press [PRGM] EDIT BULBON to edit the program listing. The variables N , representing the number of data points to collect and V , representing the time in seconds between each point are stored in the first two lines of the program for easy access. Multiply these two values and we can see that the data collected spans .08 seconds. We will modify these two variables to collect 20 data points with .008 seconds between each point. The data collected will then span .16 seconds but the longer time between points will reveal the long period behavior. Exit the program editor.

2. Now follow the experiment procedure steps 1 through 4 described in the **Discovery Experiment**.

3. After the data is collected, a plot of light intensity (in mW/cm^2) vs. time (in seconds) appears on the calculator screen. The plot should look similar to the one shown in Figure 3. Make a printout of this graph using TI-GRAPH LINK or save it as a PIC variable to be printed later. Attach this printout to your lab notebook. Be sure to include appropriate scales and axis labels on the printout. The data is saved in lists L_2 and L_4 . It would be prudent to save these lists to lists with new names, perhaps O_1 and O_2 , as subsequent experiments will erase L_2 and L_4 .

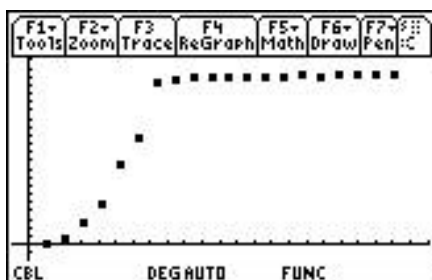


Figure 3.

6. Notice that the data shows the phenomena of the increase of light intensity without the short period variation.

Analysis and Conclusion

We will use the statistical features of the TI-89 to match the data to a logistic function.

1. The time data is stored in list L_2 and the light intensity data is stored in list L_4 . To determine the relationship between these variables, select Logistic from the [STAT] CALC menu. Enter the appropriate regression command on the home screen, Logistic L_2, L_4, Y_1 .

2. Record the regression equation and correlation coefficient in your lab notebook. Does this equation agree with the mathematical model relating intensity and time that was described in the introduction section?
3. Press [GRAPH] to see the scatter plot and regression curve together. Make a printout of this graph using TI-GRAPH LINK and attach it to your lab notebook.
4. Repeat this experiment using a different type of light source. If the source is significantly brighter, you may need to start at a separation greater than 20 in. Record all relevant data in your lab notebook, as before.
5. Truncate your data sets by deleting the last 14 points from each list (L₂ and L₄). There is no danger of losing your original data if you already saved it to new list names. Now try to fit the data to an exponential function. Select ExpReg from the [STAT] CALC menu. Enter the appropriate regression command on the home screen, ExpReg L₂, L₄, Y₁.
6. Record the regression equation and correlation coefficient in your lab notebook. Does this equation seem to indicate that the initial increase in light intensity follows an exponential curve? Can you explain this behavior from what you know about exponential functions?

EXPERIMENT 3: Bulb Glow

What is the short period behavior of the light intensity of a glowing light bulb? This experiment allows students to discover and investigate the mathematical curve that describes the steady state variation in light intensity of a glowing light bulb.

Introduction

The data collected in the **Discovery Experiment** seemed to show a short period sinusoidal variation in light intensity that appeared to ride on the overall logistic increase in light intensity. This short period phenomenon will be investigated by collecting data from a glowing light bulb and checking the fit of the data to the function

$$A \sin (Bt + C)$$

You will use the light intensity sensor to verify the relationship stated above.

Program Listing

This experiment requires that you download or enter the **BULBGLOW.89P** program listed in the appendix into your TI-89 calculator.

Experiment Procedure

1. The program **BULBGLOW** to collects short period data. The variables N , representing the number of data points to collect and V , representing the time in seconds between each point are stored in the first two lines of the program for easy access and may be modified easily if desired. We will collect 40 data points with .0005 seconds between each point. The data collected will span .02 seconds and the short time between points will reveal the short period behavior. The equipment will not handle V set to a time period shorter than .0001 seconds.

1. Place the light sensor inside the cardboard tube and position the tube so that separation between the light sensor and the center of the bulb is between 20 in. and 30 in. If the sensor is placed too close to the bulb it will become saturated with light and the intensity curve will be unnaturally flattened at the high end.

2. Darken the room and turn the light bulb on or, if you have a long tube, place the end of the tube against the light bulb to block out as much outside light as possible.

3. Make sure the CBL is turned on. Start the program **BULBGLOW** on the TI-89. The student who is positioning the light probe must carefully aim the probe at the light bulb by looking at the bulb through the tube. This position must now be held until the data is collected. If a long tube is used this care is unnecessary.

4. The program will prompt:
Press ENTER to
collect data

Hold the probe steady until the data is displayed on the TI-89.

5. After the data is collected, a plot of light intensity (in mW/cm^2) vs. time (in seconds) appears on the calculator screen. The plot should look similar to the one shown in Figure 4. Make a printout of this graph using TI-GRAPH LINK or save it as a PIC variable to be printed later. Attach this printout to your lab notebook. Be sure to include appropriate scales and axis labels on the printout. The data is saved in lists L_2 and L_4 . It would be prudent to save these lists to lists with new names, perhaps G_1 and G_2 , as subsequent experiments will erase L_2 and L_4 .

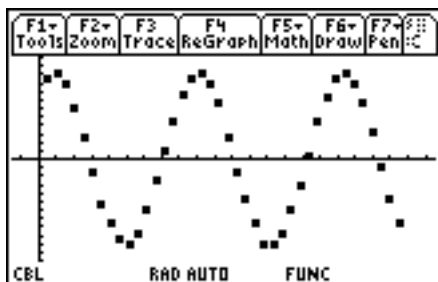


Figure 4.

6. Notice that the data shows the short period variation phenomena of the light intensity.

Analysis and Conclusion

We will use the statistical features of the TI-89 to match the data to a sine function.

1. The time data is stored in list L_2 and the light intensity data is stored in list L_4 . To determine the relationship between these variables, select SinReg from the [STAT] CALC menu. Enter the appropriate regression command on the home screen, SinReg L_2, L_4, Y_1 .
2. Record the regression equation and correlation coefficient in your lab notebook. Does this equation agree with the mathematical model relating intensity and time that was described in the introduction section? What is the source of this oscillation?
3. Press [GRAPH] to see the scatter plot and regression curve together. Make a printout of this graph using TI-GRAPH LINK and attach it to your lab notebook.
4. Repeat this experiment using a different type of light source. Is the short period behavior of the light intensity apparent in a fluorescent light?
5. Repeat the experiment by holding the light probe directly against the face of a computer monitor on a white region of the screen. What is happening with this light source? Try gathering the data again for this source with the program **BULBCOMP.89P**. Measure the length of the period by tracing along your graph from peak to peak. Now compute the frequency of the oscillation in cycles per second (Hz). Is this frequency what you might expect from the source postulated in step 2 above? Record all relevant data in your lab notebook, as before.

EXPERIMENT 4: Bulb Off

What happens when a light bulb is switched off? This experiment allows students to discover and investigate the mathematical curve that describes the light intensity decrease of a light bulb when it is turned off.

Introduction

When power is suddenly removed from the filament of the light bulb, as it is when a light is turned off the filament immediately loses its power source and quickly radiates away its remaining light energy. We would expect that the energy would dissipate in direct proportion to the remaining energy. This would give us an exponential decay in the light intensity.

$$I = A * B^t$$

We will use the TI light probe to discover the mathematical relationship of light intensity v.s. time as a light bulb decreases in brightness.

Program Listing

This experiment requires that you download or enter the **BULBOFF.89P** program listed in the appendix into your TI-89 calculator.

Experiment Procedure

1. Place the light sensor inside the cardboard tube and position the tube so that separation between the light sensor and the center of the bulb is between 20 in. and 30 in. If the sensor is placed too close to the bulb it will become saturated with light and the intensity curve will be unnaturally flattened at the high end.
2. Darken the room and turn on the light bulb or, if you have a long tube, place the end of the tube against the light bulb to block out as much outside light as possible.
3. Make sure the CBL is turned on. Start the program BULBOFF on the TI-89. The student who is positioning the light probe must carefully aim the probe at the light bulb by looking at the bulb through the tube. This position must now be held until the data is collected. If a long tube is used this care is unnecessary. When the program prompts to **PRESS [ENTER] TO ZERO** it will establish a baseline maximum light intensity.

4. The program will then prompt:
 Base Voltage = .68654
 Press ENTER to collect
 data and then turn
 the light off

5. After the data is collected, a plot of light intensity (in mW/cm^2) vs. time (in seconds) appears on the calculator screen. The plot should look similar to the one shown in Figure 5. Make a printout of this graph using TI-GRAPH LINK or save it as a PIC variable to be printed later. Attach this printout to your lab notebook. Be sure to include appropriate scales and axis labels on the printout. The data is saved in lists L_2 and L_4 . It would be prudent to save these lists to lists with new names, perhaps F_1 and F_2 , as subsequent experiments will erase L_2 and L_4 .

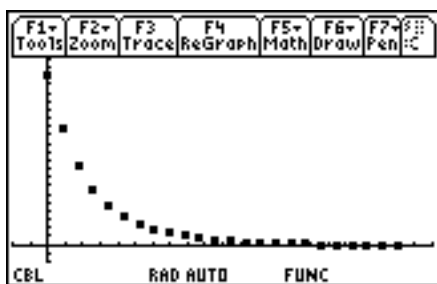


Figure 5.

Analysis and Conclusion

We will use the statistical features of the TI-89 to match the data to an exponential function.

1. The time data is stored in list L_2 and the light intensity data is stored in list L_4 . To determine the relationship between these variables, select ExpReg from the [STAT] CALC menu. Enter the appropriate regression command on the home screen, $\text{ExpReg } L_2, L_4, Y_1$.

2. Record the regression equation and correlation coefficient in your lab notebook. Does this equation agree with the mathematical model relating intensity and time that was described in the introduction section?

3. Press [GRAPH] to see the scatter plot and regression curve together. Make a printout of this graph using TI-GRAPH LINK and attach it to your lab notebook.

4. Let's experiment with the domain of the data set. Enter the command $L_2 - 4 \cdot V \rightarrow L_2$ on the home page and press [ENTER]. Now try to fit the data to the exponential function again. How does the fit compare to the original data set? Graph the data and regression curve together. Make a print out of this graph.

5. Restore your original data set by entering the commands $\downarrow F1 \rightarrow L_2$ and $\downarrow F2 \rightarrow L_4$. Now shift the data to the right by entering the command $L_2 + 4 \cdot V \rightarrow L_2$. Fit this data to the power function using $\text{PwrReg } L_2, L_4, Y_1$. Is the fit good? What is the exponent? Press [Y =] to get into the equation editor. Modify the exponent in equation Y_1 to be -2 . Now graph the resulting data set with the regression equation. Does this fit seem reasonable? Which fit is best?

EXPERIMENT 5: Computer Screen

I suppose you know that a computer screen display is made by a rapidly moving point of light that varies in color and intensity as it moves across your computer screen. Our eyes don't see the dot but blends the lines formed by its quick movement together to form the image on the screen. The number of times that the dot can redraw the screen in one second is called the refresh rate of the computer monitor. Let's see if we can measure the refresh rate of your computer monitor.

Introduction

You can probably look up the refresh rate of your monitor in the manual that came with the monitor, or maybe the rate is displayed as part of the boot sequence. If you can find this information it will give a good check of our analysis. We will use TI light probe to detect the passage of the dot across the screen and time it for one cycle, or period, T . We can then determine the refresh rate from the relationship

$$f = \frac{1}{T}$$

Program Listing

This experiment requires that you download or enter the **BULBCOMP.89P** program listed in the appendix into your TI-89 calculator.

Experiment Procedure

1. Place the front end of the light sensor right up on the computer screen in a white or otherwise brightly lit part of the screen.
2. Make sure the CBL is turned on. Start the program BULBCOMP on the TI-89.
3. The program will then prompt:
Press ENTER to
collect data
4. After the data is collected, a plot of light intensity (in mW/cm^2) vs. time (in seconds) appears on the calculator screen. The plot should look similar to the one shown in Figure 5. Make a printout of this graph using TI-GRAPH LINK or save it as a PIC variable to be printed later. Attach this printout to your lab notebook. Be sure to include appropriate scales and axis labels on the printout. The data is saved in lists L_2 and L_4 . It would be prudent to save these lists to lists with new names, perhaps F_1 and F_2 , as subsequent experiments will erase L_2 and L_4 .

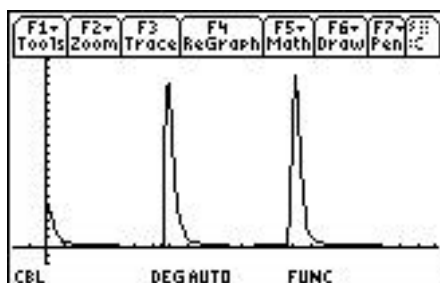


Figure 5.

Analysis and Conclusion

We will use the TRACE feature of the TI-89 to measure the time between the peaks of light intensity displayed on the graph.

1. Using [F3 TRACE] we can trace to the top of the first peak in the graph. Note the x coordinate which represents time. This is the first time the dot passed by the front of the light probe.
2. Now trace to the top of the next peak. Note this second time. The difference between the two times will be the period. Calculate the frequency by finding the inverse of the period.
3. How good is your monitor. The minimum acceptable standard for most monitors is a refresh rate of 75 Hz or better.
4. Can we get any more accuracy? If you were able to capture more than two passages of the dot then you could measure the time for a longer interval and divide by the number of intervals. This should give more accuracy.
5. Now that you have the hang of the TI light probe try it on some other light sources using the appropriate program to gather the data. For example, does the light from a fluorescent bulb have the same characteristics as light from an incandescent lamp? What are some other possibilities?

BULBON

```
Prgm
80»n
.001»v
{3, v, n, 2, 1, 1.5*q, 5}»I 1
ClrHome
ClrDraw
ClrIO
Send {6, 0}
Send {1, 0}
Send {1, 1, 1}
newList(n)»I 2
newList(n)»I 4
newList(5)»I 6
Disp "Press ENTER for baseline"
Pause
Send {3, .01, 5, 0}
Get I 6
(sum(I 6)-max(I 6)-min(I 6))/3»q
ClrIO
Disp "Base Voltage = ", q
Disp "Press ENTER to collect"
Disp "data and then turn on"
Disp "the light"
Pause
Send I 1
For i, 1, n
v*i»I 2[i]
EndFor
Get I 4
I 4-q»I 4
NewPlot 1, 1, I 2, I 4, , , , 5
ZoomData
DelVar I 1, I 6, n, i, v, q
EndPrgm
```

BULBGLOW

```
Prgm
40»n
5. -a4»v
{3, v, n, 0}»I 1
Cl rHome
Cl rDraw
Cl rI 0
Send {6, 0}
Send {1, 0}
Send {1, 1, 1}
newLi st(n)»I 2
newLi st(n)»I 4
Di sp "Press ENTER to"
Di sp "col lect data"
Pause
Send I 1
For i, 1, n
v*i »I 2[i]
EndFor
Get I 4
(max(I 4)-mi n(I 4))/2+mi n(I 4)»q
I 4-q»I 4
. 001»xscl
. 01»yscl
NewPl ot 1, 1, I 2, I 4, , , , 4
ZoomData
Del Var I 1, n, i, v, q
EndPrgm
```

BULBOFF

```
Prgm
25»n
.008»v
{3, v, n, 3, 1, .5*q, 12}»I 1
ClrHome
ClrDraw
ClrIO
Send {6, 0}
Send {1, 0}
Send {1, 1, 1}
newList(n)»I 2
newList(n)»I 4
newList(5)»I 6
Disp "Press ENTER for baseline"
Pause
Send {3, .01, 5, 0}
Get I 6
(sum(I 6)-max(I 6)-min(I 6))/3»q
ClrIO
Disp "Base Voltage = ", q
Disp "Press ENTER to collect"
Disp "data and then turn"
Disp "the light off"
Pause
Send I 1
For i, 1, n
v*i»I 2[i]
EndFor
Get I 4
I 4-min(I 4)»I 4
NewPlot 1, 1, I 2, I 4, , , , 4
ZoomData
DelVar I 1, I 6, n, i, v, q
EndPrgm
```

BULBCOMP

```
Prgm
150»n
0.0002»v
{3, v, n, 0}»l 1
ClrHome
ClrDraw
ClrIO
Send {6, 0}
Send {1, 0}
Send {1, 1, 1}
newList(n)»l 2
newList(n)»l 4
Disp "Press ENTER to"
Disp "collect data"
Pause
Send l 1
For i, 1, n
v*i»l 2[i]
EndFor
Get l 4
min(l 4)»q
l 4-q»l 4
NewPlot 1, 2, l 2, l 4, , , , 5
0.001»xscl
0.01»yscl
ZoomData
DelVar l 1, n, i, v, q
EndPrgm
```